Terminal Velocity of a Laser-Driven Light Sail

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It is shown that the maximum velocity attainable by a laser-driven light sail is constrained by the back pressure of the relativistically transformed microwave background radiation field with interstellar material having little effect. Using current sail concepts, terminal Lorentz factors much in excess of 10^2 are not possible, and therefore, limitations are imposed on the benefits of relativistic time dilation and the possible use of such sails for travel over long interstellar distances. This is a fundamental constraint imposed by nature on the ultimate performance of such sails.

Nomenclature

A (dA)	= sail area (area element)
c	= velocity of light, 3×10^8 ms^{-1}
E, E_L	= energy flux/laser energy output
$F(\cos\theta, \beta)$	= function, Eq. (14)
h	= Planck constant, 6.63×10^{-34} Js^{-1}
K	= proton energy loss per unit path length in sail material
l	= sail thickness
m_0	= hydrogen rest mass, 1.67×10^{-27} kg
n	= number density, photon/particle
$\boldsymbol{n}$	= direction vector of photon
N	= absolute number, photon/particle
p	= momentum, photon/particle
P	= pressure, radiation/interstellar medium
R	= ratio of pressures from ambient radiation and matter
t (dt)	= time (time element)
T	= radiation field temperature
U	= energy density
v	= sail velocity
V (dV)	= volume (volume element)
W_0, W_s	= laser power/power density
$x^{1,2,3}$	= sail reference frame
β	= normalized sail velocity, v/c
γ	= Lorentz factor, $(1-\beta^2)^{-1/2}$
Δ	= difference in two variables
ϵ	= photon energy-momentum four-vector
θ	= polar angle of photon
Λ	= Lorentz transformation
$\lambda, \lambda_{1,2}$	= dimensionless parameters
ν	= photon frequency
ξ	= fractional transfer of particle momentum to sail
ρ	= mass density of the interstellar medium
σ	= Stefan-Boltzmann constant 5.67×10^{-8} Wm^{-2} K^{-4}
Σ	= sail surface density
T	= particle kinetic energy
ϕ	= photon azimuthal angle
Ω ($d\Omega$)	= solid angle (solid-angle element)

Superscripts

$'$	= moving reference frame
$*$	= final value

Subscripts

H	= interstellar hydrogen
0	= proper (rest frame) value
∞	= terminal value
ν	= per unit frequency
RAD	= ambient radiation field
ISM	= interstellar medium
LASER	= driving laser

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Introduction

THE concept of the laser-driven light sail was first discussed by Forward[1] in 1962 as a novel method of interstellar travel. It was proposed that large solar-pumped lasers be used to create a single collimated laser beam to drive a lightweight reflective sail by radiation pressure over interstellar distances in reasonably short time scales. This idea was independently developed by Marx[2] in 1966 and followed up by Redding.[3] More recently Forward[4] in 1984 gave a detailed account of a feasible system that allowed roundtrip travel to the nearest stars well within a human lifetime. In this concept the sail was divided into three concentric segments with two outer rings surrounding a smaller central return sail. On arrival at the target star the outer ring of the sail would separate and reflect laser light back onto the inner two segments of the sail, thus decelerating them and allowing exploration of the target star system. To make the return journey the second of the rings would be used to reflect the laser light onto and accelerate the central return sail, which would be decelerated directly by the laser on arrival at the solar system.

In this paper it is shown that such sails have a terminal velocity imposed by the back pressure of the relativistically transformed 2.75 K microwave background radiation field, limiting their performance and possible use for travel over large interstellar distances. In the first section a rest frame analysis of the dynamics of the system is carried out, the results of which are carried over into a relativistic analysis where the back pressure of the relativistically transformed microwave background radiation field and of reradiation by the sail are calculated along with the relativistic transformation of the laser-induced radiation pressure. Using these results a simple one-dimensional equation of motion may be formed allowing the calculation of the sail terminal velocity. A comparison is then made of the effect of the back pressure of the interstellar medium to that of the microwave background radiation field, and it is found that the interstellar medium is of considerably less importance than the microwave background radiation field because of the extremely low energy losses of particles passing through the sail at relativistic velocities. Finally, an analysis of the heating of the sail by ambient radiation and matter is carried out, and a comparison is made with the sail heating caused by the incident laser energy.

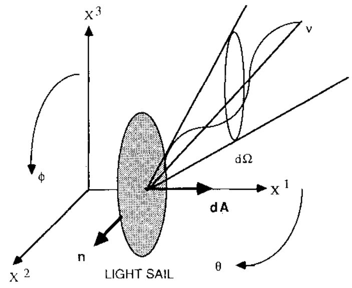

Fig. 1 Sail at rest with respect to the ambient radiation field.

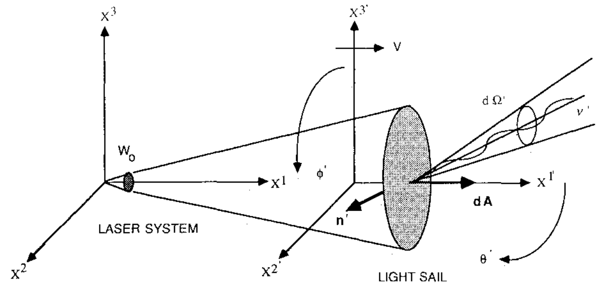

Fig. 2 Sail moving with a velocity v through the ambient radiation field.

Rest Frame Analysis

In this first section we will calculate the net radiation pressure on a sail at rest in a frame (x^1,x^2,x^3) due to the microwave background radiation field. This rest frame is defined to be the frame in which the radiation field has an isotropic temperature as measured from the sail. Consider now a sail surface element $\mathbf{d}A$ at rest with respect to an ambient radiation field of energy density U_0 with a photon of frequency ν incident along a direction $\boldsymbol{n}$ onto the sail surface element (Fig. 1). This photon transfers an amount of momentum Δp normal to the sail element where

$$\Delta p = \frac{h\nu}{c}\frac{\boldsymbol{n}\cdot\mathbf{d}A}{|\mathbf{d}A|} \tag{1}$$

Photons arriving from direction $\boldsymbol{n}$ in solid angle $\mathrm{d}\Omega$ and crossing sail element $\mathbf{d}A$ in time $\mathrm{d}t$ must be contained within a volume element $\mathrm{d}V$ given by

$$\mathrm{d}V = c\,\mathrm{d}t\,\mathbf{d}A\left|\frac{\boldsymbol{n}\cdot\mathbf{d}A}{|\mathbf{d}A|}\right| \tag{2}$$

Therefore, since the radiation field is isotropic in the rest frame, the number of photons in the frequency range $(\nu,\nu+\mathrm{d}\nu)$ arriving along direction $\boldsymbol{n}$ in solid angle $\mathrm{d}\Omega$ incident on sail element $\mathbf{d}A$ in time $\mathrm{d}t$ is given by

$$\mathrm{d}N_\nu = \frac{\mathrm{d}\Omega}{4\pi}\,n_\nu c\,|\cos\theta|\;\mathrm{d}\nu\,\mathrm{d}t\,\mathbf{d}A \tag{3}$$

where n_ν is the photon number density in this frequency range. Hence, using Eq. (1), the momentum transfer normal to the sail element is then found to be

$$\mathrm{d}p_\nu = \frac{U_\nu}{4\pi}\cos\theta\,|\cos\theta|\;\mathrm{d}\nu\,\mathrm{d}t\,\mathbf{d}A\,\mathrm{d}\Omega \tag{4}$$

where $U_\nu = h\nu n_\nu$ is the photon energy density per unit frequency range. Integrating over all frequencies we obtain the pressure on the sail due to photons incident from solid angle $\mathrm{d}\Omega$:

$$\mathrm{d}P_0 = \frac{U_0}{4\pi}\cos\theta\,|\cos\theta|\;\mathrm{d}\Omega \tag{5}$$

with $\mathrm{d}\Omega = \sin\theta\mathrm{d}\theta\mathrm{d}\phi$ and U_0 the frequency integrated energy density.

For ease of illustration we will assume the sail has a perfectly reflecting back surface ($-x^1$ direction) and a perfectly absorbing front surface ($+x^1$ direction). Then, since the front surface is now being considered as a perfect blackbody absorber, it will also be a perfect emitter so that for thermodynamic consistency we must also take account of the radiation isotropically re-emitted in the sail frame from the front surface of the sail. Furthermore, since the sail will now be in thermodynamic equilibrium with the ambient radiation field, the re-emitted radiation will be equal in pressure to that being absorbed. Therefore, to obtain the net radiation pressure on the sail due to the ambient radiation field, we integrate Eq. (5) over all solid angle with the appropriate contributions to the net radiation pressure from each side of the sail, viz.,

$$P_0 = \frac{U_0}{2\pi}\left(\int_0^{2\pi}\int_{\pi/2}^{\pi}\cos\theta^2\,\sin\theta\mathrm{d}\theta\mathrm{d}\phi - \int_0^{2\pi}\int_0^{\pi/2}\cos\theta^2\,\sin\theta\mathrm{d}\theta\mathrm{d}\phi\right) \tag{6}$$

Hence, since the two integrals in Eq. (7) are odd, the net radiation pressure on the sail in the rest frame is zero, with the radiation pressure on each side of the sail being equal and given by

$$P_0 = \frac{4\sigma}{3c}\,T_0^4 \tag{7}$$

where, for a blackbody radiation field of ambient temperature T_0, $U_0 = (4\sigma/c)T_0^4$. It is interesting to note that if the sail were initially at a temperature below 2.75 K and so not in thermodynamic equilibrium with the ambient radiation field, a fraction of the radiation incident on the front surface would be absorbed and not re-emitted, so that the sail would self-accelerate due to the differential radiation pressure across its surfaces. This self-acceleration would, however, terminate as soon as the sail achieved thermodynamic equilibrium with its surroundings.

Relativistic Analysis

In order to discuss the motion of the sail at high velocities we must now develop a relativistic approach to the problem. Consider now the sail in a primed frame $(x^{1'},x^{2'},x^{3'})$ moving with respect to the unprimed frame (x^1,x^2,x^3) with a velocity v directed along the coincident x^1 and $x^{1'}$ axes with a relativistically transformed radiation field now being observed (Fig. 2). The energy-momentum four-vector of a photon of frequency ν incident on the sail along a direction $\boldsymbol{n}$ in the unprimed is given by

$$\epsilon = \frac{-h\nu}{c}\begin{Bmatrix} 1 \\ \cos\theta \\ \sin\theta \quad \sin\phi \\ \sin\theta \quad \cos\phi \end{Bmatrix} \tag{8}$$

which will transform contravariantly into the primed frame as

$$\epsilon^{k'} = \Lambda_j^{k'}\,\epsilon^j \tag{9}$$

where the matrix $\Lambda_j^{k'}$ is the proper Lorentz transformation ($\det\Lambda = +1$) for motion along the x^1 axis given by

$$\Lambda_j^{k'} = \begin{Bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Bmatrix}, \qquad \beta = v/c, \quad \gamma = (1-\beta 2)^{-1/2} \tag{10}$$

Solving the system of four scalar equations resulting from the matrix equation simultaneously, it is found that the only independent solutions are

$$\nu' = \nu \frac{1 + \cos\theta\beta}{(1-\beta^2)^{1/2}}, \quad \cos\theta' = \frac{\cos\theta + \beta}{1 + \cos\theta\beta}, \quad \phi' = \phi \tag{11}$$

describing the aberration and Doppler shift of the incident photons in the primed frame.

Following the rest frame analysis we may write the momentum transfer in the sail frame normal to the sail element dA due to photons in the frequency range $(\nu', \nu' + d\nu')$ incident along direction $\boldsymbol{n}'$ in solid angle $d\Omega'$ in time dt as given by

$$dp'_{\nu'} = \frac{U_{\nu'}}{4\pi} \cos\theta' |\cos\theta'| \, d\nu' dt dA \, d\Omega' \tag{12}$$

where, for a blackbody radiation field, the energy density $U_{\nu'}$ is given by the Planck function

$$U_{\nu'}(T') = \frac{8\pi}{3c^3} \frac{h\nu'^3}{\exp(h\nu'/kT') - 1} \tag{13}$$

Furthermore, from the invariance of phase space density under a Lorentz transformation, it may be shown that the quantity $(U_{\nu'}/\nu'^3)$, defined by Eq. (13), is a Lorentz invariant,[5] allowing the transformation of the radiation field energy density to be found. Using this fact and the variable transformations defined by Eq. (11) we may integrate Eq. (12) over all frequencies to obtain the net radiation pressure on the sail in the primed frame dP_1' due to photons from solid-angle element $d\Omega$ in the unprimed frame where

$$\begin{aligned} dP_1' &= \frac{U_0}{4\pi} \frac{\cos\theta + \beta}{1 + \cos\theta\beta} \left| \frac{\cos\theta + \beta}{1 + \cos\theta\beta} \right| \frac{(1 + \cos\theta\beta)^2}{(1 - \beta^2)} d\Omega \\ &= \frac{U_0}{4\pi} F(\cos\theta, \beta) \, d\Omega \end{aligned} \tag{14}$$

As in the rest frame analysis we will, for ease of illustration, assume the sail has a perfectly reflecting rear surface and a perfectly absorbing, and therefore, perfectly emitting front surface. To obtain thermodynamic consistency we again must take account of the radiation pressure due to the absorbed energy from the relativistically transformed ambient radiation field being isotropically re-emitted in the rest frame of the sail. This absorbed energy may be calculated by considering the energy from the number flux of photons incident on the front surface of the sail in the primed frame, cf. Eq. (3) in the rest frame:

$$dE' = \frac{U_{\nu'}}{4\pi} c |\cos\theta'| \, d\nu' \, dt \, dA \, d\Omega' \tag{15}$$

Using the previously defined transformations and integrating over the solid angle containing the photons incident on the front absorbing surface of the sail, we obtain the energy absorbed per unit sail area per unit time by the front surface of the sail from the relativistically transformed ambient radiation field, viz.,

$$E'(\beta) = (1/4) U_0 c \, \gamma^2 [\beta^2 + (8/3)\beta + 1] \tag{16}$$

which must be equal to the energy re-emitted from the unit sail area in unit time since the front surface of the sail is also a perfect blackbody emitter. Additionally, for a blackbody radiator, the energy density in the blackbody is related to the emergent energy flux by $U' = 4E'/c$ so that we may now calculate, using Eq. (5), the radiation pressure dP_2' due to the isotropic re-emission in the rest frame of the sail of the absorbed radiation:

$$dP_2' = \frac{-U_0}{4\pi} \gamma^2 \left(\beta^2 + \frac{8}{3}\beta + 1 \right) \cos\theta |\cos\theta| \, d\Omega \tag{17}$$

Integrating Eq. (17) over the solid angle containing the isotropically re-emitted photons we obtain

$$\begin{aligned} P_2'(\beta) &= \frac{-U_0}{4\pi} \gamma^2 \left(\beta^2 + \frac{8}{3}\beta + 1 \right) \int_0^{2\pi} \int_0^{\pi/2} \cos\theta^2 \sin\theta d\theta d\phi \\ &= -\frac{1}{6} U_0 \gamma^2 \left(\beta^2 + \frac{8}{3}\beta + 1 \right) \end{aligned} \tag{18}$$

To obtain the radiation pressure on the sail due to photons absorbed on the front surface and perfectly reflected on the rear surface we must now integrate Eq. (14) over all solid angle with the appropriate contributions to the net radiation pressure from each side of the sail:

$$\begin{aligned} P_1'(\beta) &= \frac{U_0}{4\pi} \left(2 \int_0^{2\pi} \int_{\pi/2}^{\pi} F(\cos\theta, \beta) \sin\theta \, d\theta \, d\phi \right. \\ &\quad \left. - \int_0^{2\pi} \int_0^{\pi/2} F(\cos\theta, \beta) \sin\theta \, d\theta \, d\phi \right) \\ &= \frac{1}{6} U_0 \gamma^2 (3\beta^2 - 9\beta + 1) \end{aligned} \tag{19}$$

Thus, the net radiation pressure on a sail with a perfectly reflecting rear surface and perfectly absorbing front surface moving at velocity v through a blackbody radiation field of proper energy density U_0 is $P'(\beta)_{\mathrm{RAD}}$ given by $P_1'(\beta) + P_2'(\beta)$, viz.,

$$P'(\beta)_{\mathrm{RAD}} = (1/3) U_0 \, \gamma^2 \beta [2\beta - (35/6)] \tag{20}$$

At $\beta = 0$ it is seen that the net radiation pressure on the sail from the ambient radiation field is zero because of the isotropy of the incident radiation. However, in the asymptotic limit as $\beta \to 1$, we see that $P'(\beta)_{\mathrm{RAD}} \to -\infty$ due to the increasingly aberrated and blue-shifted radiation in front of the sail and correspondingly red-shifted radiation behind. Thus, at relativistic velocities the sail will experience a large back pressure due to the relativistically transformed ambient radiation field opposing the sail motion.

Finally, for the case of a laser-driven sail, we must calculate how the laser-induced radiation pressure will transform. A laser of power W_0 will exert a radiation pressure P_{LASER} on the rear, reflective surface of the sail in the rest frame, where

$$P_{\mathrm{LASER}} = \frac{2W_0}{c} \tag{21}$$

Now, in the sail frame, the apparent laser power is W' given by

$$W' = \left\{ \frac{dE_L'}{dt'} \right\} = W_0 \left\{ \frac{dE_L'}{dE_L} \right\} \left\{ \frac{dt}{dt'} \right\} \tag{22}$$

However, the laser energy output and time element transform as

$$\frac{dE_L'}{dE_L} = \gamma(1-\beta), \qquad \frac{dt'}{dt} = \gamma(1+\beta) \tag{23}$$

where $\gamma v dt$ is the additional path length the laser photons must traverse in time dt due to the motion of the sail. Hence, the laser-induced radiation pressure transforms as

$$P'(\beta)_{\mathrm{LASER}} = P_{\mathrm{LASER}} \left\{ \frac{1-\beta}{1+\beta} \right\} \tag{24}$$

Thus, as the sail is accelerated by the laser system, the driving pressure on the sail will decrease, as described by Eq. (24), and the back pressure due to the ambient radiation field will increase, as described by Eq. (20), so that we would expect the sail to attain some terminal velocity when these two pressures equate.

Laser-Driven Sail Terminal Velocity

Using the transformed radiation pressures calculated in the previous sections we now calculate the maximum velocity attainable by the sail. From Eqs. (20) and (24) we may write down a one-dimensional equation of motion for unit area of a laser-driven sail of surface density Σ_0 ($\mathrm{kgm^{-2}}$) and incident laser power density W_s ($\mathrm{Wm^{-2}}$) moving through a blackbody radiation field of proper energy density U_0. In the frame of the sail we have

$$\Sigma_0 c \frac{\mathrm{d}\beta}{\mathrm{d}t} = \frac{2W_s}{c}\left[\frac{1-\beta}{1+\beta}\right] + \frac{1}{3}U_0\gamma^2\beta\left(\beta - \frac{35}{6}\right) \tag{25}$$

Thus, as $t \to \infty$, the sail will reach a terminal velocity of β_∞ when $(\mathrm{d}\beta/\mathrm{d}t) = 0$, where β_∞ is the *physical* root of the quadratic resulting from Eq. (25):

$$(\lambda + 1)\beta_\infty^2 - [(35/6)\lambda + 2]\beta_\infty + 1 = 0 \tag{26}$$

and $\lambda = (U_0 c/6W_s)$ is a convenient dimensionless parameter. Hence, solving Eq. (26), we obtain an exact expression for the sail terminal velocity in terms of the parameter λ:

$$\beta_\infty = \frac{1}{2}\left[\frac{2 + (35/6)\lambda}{1+\lambda}\right] - \frac{[(35\lambda/6)^2 + (23/3)\lambda]^{1/2}}{2(1+\lambda)} \tag{27}$$

and for $\lambda \ll 1$ we may write as an approximation, to the lowest order in λ,

$$\beta_\infty \simeq \left\{1 - \left[\frac{23}{12}\frac{cU_0}{W_s}\right]^{1/2}\right\} \tag{28}$$

As an example, consider now a sail being driven through interstellar space with a typical incident power density of $1 \times 10^4\ \mathrm{Wm^{-2}}$ in the rest frame.[4] The 2.75 K microwave background radiation field has a proper energy density of $4.32 \times 10^{-14}\ \mathrm{Jm^{-3}}$ so that the sail terminal velocity is $\beta_\infty = 0.9999718933$, as given by Eq. (27). Even though this velocity is close to that of light, the corresponding Lorentz factor is $\gamma_\infty = 133.4$, so that relativistic time dilation is constrained by the presence of the microwave background radiation field as is the distance that such a sail may travel in a reasonable fraction of a human lifetime. There are, of course, regions of space with ambient radiation fields of considerably greater energy density than that of the microwave background field. For example, the radiation field energy density toward the galactic center is $1.60 \times 10^{-11}\ \mathrm{Jm^{-3}}$, giving a terminal Lorentz factor of only $\gamma_\infty = 35.7$, so that the presence of ambient radiation also restricts the *regions* of space through which such sails may travel at highly relativistic velocities.

Pressure of the Interstellar Medium

Approximately 99% of interstellar matter is in the form of monatomic hydrogen, with the remaining 1% as dust with grain sizes of order 0.5 μm. Recent studies[6] show that this matter has a density in the local region of 10^{-22} to 10^{-24} $\mathrm{kgm^{-3}}$. From the sail frame, interstellar matter will intercept the sail with relativistic velocities and corresponding energies ($\simeq$ 125 GeV at the previously calculated terminal velocity), giving rise to a further back pressure opposing the sail motion. The relativistic momentum of an interstellar hydrogen atom may be written as

$$p_H = m_0 c(\gamma^2 - 1)^{1/2} \tag{29}$$

with the apparent number density of these particles being γn_0, where n_0 is their proper number density. Hence, the flux of particles across the sail is given by

$$N_H = \gamma\, n_{H0}\beta c \tag{30}$$

However, only a small fraction ξ of the particle momentum will be transferred to a sufficiently thin sail, so that the back pressure on the sail due to interstellar hydrogen is given by

$$P'(\beta)_{\mathrm{ISM}} = -\rho_0 c^2\,\gamma^2\beta^2\,\xi \tag{31}$$

where ρ_0 is the proper mass density of the interstellar medium. This term may be added to Eq. (25) to give a modified equation of motion, viz.,

$$\Sigma_0 c \frac{\mathrm{d}\beta}{\mathrm{d}t} = \frac{2W_s}{c}\left[\frac{1-\beta}{1+\beta}\right] + \gamma^2\left[\frac{1}{3}U_0\beta\left(2\beta - \frac{35}{6}\right) - \rho_0 c^2\beta^2\xi\right] \tag{32}$$

The terminal velocity can now be calculated as the *physical* root of the quadratic

$$\lambda_1\,\beta_\infty^2 - \lambda_2\,\beta_\infty + 1 = 0 \tag{33}$$

where the coefficients are now of the form

$$\lambda_1 = 1 + \frac{U_0 c}{6W_s} - \frac{\rho_0 c^3\xi}{2W_s} \tag{34a}$$

$$\lambda_2 = 2 + \frac{35}{36}\frac{U_0 c}{W_s} \tag{34b}$$

The value of ξ may be obtained from the energy loss per unit path length for a given sail material and sail thickness. A particle of initial kinetic energy T_H incident on a sail of thickness l and an energy loss per unit path length of $(\mathrm{d}E/\mathrm{d}X) = K$ will have a kinetic energy of T_H^* after passing through the sail where

$$T_H^* \simeq T_H - Kl = m_0c^2(\gamma^* - 1) \tag{35}$$

Therefore, the change in the particle Lorentz factor is approximately

$$\Delta\gamma \simeq \frac{Kl}{m_0c^2} \tag{36}$$

Furthermore, we may write for the change in particle momentum

$$\Delta p = m_0 c\ \gamma(\gamma^2 - 1)\ \Delta\gamma \tag{37}$$

Thus, if we write $\xi \simeq (\Delta p/p)$ it is found that

$$\xi(\beta) \simeq \beta(1 - \beta^2)^{1/2}(Kl/m_0c^2) \tag{38}$$

where $\xi \to 0$ as $\beta \to 1$, i.e., the sail becomes transparent to interstellar hydrogen at high energies, as noted by Rather.[7]

Consider now the ratio of radiation pressure from the ambient radiation field to that of the interstellar medium where, from Eqs. (20) and (31),

$$R = \frac{U_0}{6\rho_0 c^2\xi}\left[\frac{35}{3\beta} - 2\right] \tag{39}$$

In the relativistic limit of $\beta \to 1$, R is finite and is given by

$$R \to \frac{29\,U_0}{18\rho_0 c^2\,\xi} \tag{40}$$

For energies in the low gigaelectron volt range, where accurate experimental data is available, the energy loss per unit path length for protons incident on aluminum is approximately 450 $\mathrm{MeVm^{-1}}$. Thus, for a typical aluminum sail of 10-nm thickness, $\xi \simeq 3 \times 10^{-11}$, and taking the local density of the interstellar medium to be the upper limit of $10^{-22}\ \mathrm{kgm^{-3}}$, we find that $R \simeq 260$ in this energy range. It would seem then that, in fact, interstellar hydrogen has little effect on thin light sails due to the small values of $(\mathrm{d}E/\mathrm{d}X)$ at high incident particle energies so that the radiation pressure from the microwave background radiation field will dominate.

It is difficult to model the effect of interstellar dust grains in detail but, due to their extremely low mass density, it is clear that their direct pressure on the sail will be negligible, although they will cause local damage to the sail. Any micrometer scale perforations in the sail due to interstellar dust grains will not affect the sail reflective properties so long as the perforation scale remains less than the wavelength of the laser light used. Also, a simple calculation shows that the time scale for any noticeable degradation of the total sail surface area will be many times the period of high-velocity sail flight.

Sail Heating

In a somewhat more realistic case where the sail is not perfectly reflecting and absorbs a fraction of the incident laser energy we must consider the effects of sail heating. As well as carrying momentum, the photons from the ambient radiation field also carry energy, which will be transferred to the sail. The sail must then reradiate this energy along with the energy absorbed from the laser itself, which is of the order of 10^3 Wm^{-2} in the initial nonrelativistic stages of the flight.

From Eq. (16) we have the energy absorbed from the ambient radiation field by the front surface of the sail:

$$E'(\beta)_{\mathrm{RAD}} = (1/4)U_0 c\, \gamma^2\, [\beta^2 + (8/3)\beta + 1] \tag{41}$$

At the previously calculated terminal velocity β_∞, the energy flux is found to be only 2×10^{-3} Wm^{-2}, which, even if the front surface is assumed to be a perfect absorber, is negligible in comparison to the energy flux that must be absorbed and reradiated by the sail from the laser in the initial stages of the flight. The energy flux due to kinetic energy losses from interstellar hydrogen passing through the sail may also be calculated from the energy loss per unit path length of the material. Each hydrogen atom will lose an amount of kinetic energy ΔT_H

$$\Delta T_H \simeq \frac{\mathrm{d}E}{\mathrm{d}X}\, l \tag{42}$$

Thus, with particle flux across the sail given by Eq. (30), the energy flux is then

$$E'(\beta)_{\mathrm{ISM}} = \frac{\rho_0}{m_0} Kcl\, \gamma\beta \tag{43}$$

For the previously chosen numerical values, the energy flux from the interstellar medium at β_∞ is found to be only 3×10^{-3} Wm^{-2}. Again, this is a negligible amount when compared to the maximum energy flux absorbed from the laser. It is seen then that the sail terminal velocity is not constrained by the energy losses of the ambient radiation and matter to the sail but only by the momentum losses, particularly from the microwave background radiation field.

Conclusions

The back pressure on a laser-driven light sail traveling relativistically through the microwave background radiation field has been calculated and compared with that from the interstellar medium. It was found that the major factor limiting the performance of such a sail is, in fact, the ambient radiation field since the sail becomes transparent to the interstellar medium at sufficiently high sail velocities. Although for a sail in interstellar space the terminal velocity is close to the velocity of light, the corresponding Lorentz factor is limited to an order of 10^2, thereby constraining the relativistic time dilation attainable in the sail frame and therefore the distance that may be traveled by the sail in a reasonable fraction of a human lifetime.

A possible solution to the problem would be to use a long, conical sail pointing in the direction of motion, which would obliquely reflect photons incident from the aberrated ambient radiation incident from the forward direction and thus reduce the component of photon momentum transferred normal to the sail. However, from the rest frame, the cone would appear shortened at relativistic velocities due to Lorentz contraction so that the cone length would have to be many times its diameter to be of use. By using this sail geometry, higher terminal velocities and Lorentz factors would be attainable, but at the cost of the large increase in sail mass and corresponding reduction in sail acceleration for a given laser power.

Compared with other proposals for fast interstellar travel such as fusion and antimatter schemes, the light sail does not have the same problems with erosion by the interstellar medium due to its extremely small mass to frontal area ratio, allowing particles to pass through the sail almost unimpeded. The sail performance is limited only by the presence of the microwave background radiation field and how far a coherent and collimated laser beam may be projected across space. For these reasons, the laser-driven light sail appears to be one of the most promising and technologically feasible means of traveling to the stars.

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